

Lepton and Quark Mass Matrices

H. NISHIURA

*Department of General Education, Junior College of Osaka Institute of Technology,
Asahi-ku, Osaka, 535-8585 Japan*

K. MATSUDA and T. FUKUYAMA

Department of Physics, Ritsumeikan University, Kusatsu, Shiga, 525-8577 Japan

(February 17, 1999)

Abstract

We propose a model that all quark and lepton mass matrices have the same zero texture. Namely their (1,1), (1,3) and (3,1) components are zeros. The mass matrices are classified into two types I and II. Type I is consistent with the experimental data in quark sector. For lepton sector, if seesaw mechanism is not used, Type II allows a large ν_μ - ν_τ mixing angle. However, severe compatibility with all neutrino oscillation experiments forces us to use the seesaw mechanism. If we adopt the seesaw mechanism, it turns out that Type I instead of II can be consistent with experimental data in the lepton sector too.

PACS number(s): 12.15Ff, 14.60Gh, 14.80Dq

Typeset using REVTeX

One of the ultimate goals in particle physics is to construct the unified model of quarks and leptons. Phenomenological construction of quark and lepton mass matrices can be an important step toward this goal, which reproduces and predicts direct and indirect observed quantities like quark and lepton masses, mixing angles and CP violating phases. In this paper we propose a model that all quark and lepton mass matrices, M_u , M_d , M_ν and M_e , (mass matrices of up quarks (u, c, t) , down quarks (d, s, b) , neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ and charged leptons (e, μ, τ) , respectively) have the same zero texture [1]. Here $M_\nu = -M_D^T M_R^{-1} M_D$ is the mass matrix of light Majorana neutrinos, which is considered to be constructed via the seesaw mechanism [2] from the neutrino mass matrix,

$$\begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix}, \quad (1)$$

where M_D is the Dirac neutrino mass matrix and M_R is the Majorana mass matrix of the right-handed components. M_D and M_R are furthermore assumed to have the same zero texture matrix as M_ν . This assumption restricts the texture forms as follows.

$$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \quad \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \quad \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}, \quad \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix},$$

$$\begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix}, \quad \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}, \quad \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}. \quad (2)$$

Here $*$'s indicate suitable nonzero numbers. Among these forms we choose the first one because it is most close to the NNI form [3] in which (2,2) component is also zero. Namely, our texture of mass matrix is

$$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}. \quad (3)$$

Indeed, this matrix leaves its form in the seesaw mechanism as

$$\underbrace{\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}}_{M_D^T} \underbrace{\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}}_{M_R^{-1}} \underbrace{\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}}_{M_D} = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}. \quad (4)$$

The nonvanishing (2,2) component distinguishes our form from NNI. This difference, as will be shown, makes it possible to treat quark and lepton mass matrices universally and consistently with experiments.

Now we assign quark and lepton mass matrices as follows.

$$\begin{aligned} M_u &= \begin{pmatrix} 0 & A_u & 0 \\ A_u & B_u & C_u \\ 0 & C_u & D_u \end{pmatrix}, & M_\nu &= \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu \end{pmatrix}, \\ M_d &= P_d \begin{pmatrix} 0 & A_d & 0 \\ A_d & B_d & C_d \\ 0 & C_d & D_d \end{pmatrix} P_d^\dagger = \begin{pmatrix} 0 & A_d e^{i\alpha_{12}} & 0 \\ A_d e^{-i\alpha_{12}} & B_d & C_d e^{i\alpha_{23}} \\ 0 & C_d e^{-i\alpha_{23}} & D_d \end{pmatrix}, \\ M_e &= P_e \begin{pmatrix} 0 & A_e & 0 \\ A_e & B_e & C_e \\ 0 & C_e & D_e \end{pmatrix} P_e^\dagger = \begin{pmatrix} 0 & A_e e^{i\beta_{12}} & 0 \\ A_e e^{-i\beta_{12}} & B_e & C_e e^{i\beta_{23}} \\ 0 & C_e e^{-i\beta_{23}} & D_e \end{pmatrix}. \end{aligned} \quad (5)$$

where $P_d \equiv \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$, $\alpha_{ij} \equiv \alpha_i - \alpha_j$, and $P_e \equiv \text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$, $\beta_{ij} \equiv \beta_i - \beta_j$.

Let us discuss the relations between the following texture's components of mass matrix M ,

$$M = \begin{pmatrix} 0 & A & 0 \\ A & B & C \\ 0 & C & D \end{pmatrix} \quad (6)$$

and its eigen mass m_i . They satisfy

$$B + D = m_1 + m_2 + m_3,$$

$$\begin{aligned}
BD - C^2 - A^2 &= m_1 m_2 + m_2 m_3 + m_3 m_1, \\
DA^2 &= -m_1 m_2 m_3.
\end{aligned} \tag{7}$$

Therefore, mass matrix is classified into two types by choosing B and D as follows:

$$\begin{aligned}
[\text{Type I}](B:\text{large}) \quad & B = m_2, \quad D = m_3 + m_1 \\
[\text{Type II}](B:\text{small}) \quad & B = m_1, \quad D = m_3 + m_2
\end{aligned} \tag{8}$$

Here we don't accept the case of $B = m_1 + m_2$ and $D = m_3$ since in this case C becomes zero and this matrix is out of our texture any more. We adopt Type I for quark mass matrices. For lepton sector we adopt Type I and Type II mass matrices for the case with and without seesaw mechanism, respectively. We proceed to discuss in detail.

Let us discuss the quark sector first. The mass matrices of Type I ($B = m_2, D = m_3 + m_1$) explains the quark sector consistently as will be shown. Assigning a definite value $B = m_2$ and $D = m_3 + m_1$ in (7) for Type I, we obtain

$$A = \sqrt{\frac{(-m_1)m_2m_3}{m_3 + m_1}}, \quad C = \sqrt{\frac{(-m_1)m_3(m_3 - m_2 + m_1)}{m_3 + m_1}}. \tag{9}$$

Then mass matrix of Type I becomes

$$M = \begin{pmatrix} 0 & \sqrt{\frac{m_1 m_2 m_3}{m_3 - m_1}} & 0 \\ \sqrt{\frac{m_1 m_2 m_3}{m_3 - m_1}} & m_2 & \sqrt{\frac{m_1 m_3 (m_3 - m_2 - m_1)}{m_3 - m_1}} \\ 0 & \sqrt{\frac{m_1 m_3 (m_3 - m_2 - m_1)}{m_3 - m_1}} & m_3 - m_1 \end{pmatrix} \simeq \begin{pmatrix} 0 & \sqrt{m_1 m_2} & 0 \\ \sqrt{m_1 m_2} & m_2 & \sqrt{m_1 m_3} \\ 0 & \sqrt{m_1 m_3} & m_3 - m_1 \end{pmatrix} \tag{10}$$

(for $m_3 \gg m_2 \gg m_1$).

Here we have transformed m_1 into $-m_1$ by rephasing. M is diagonalized by an orthogonal matrix O as

$$O^T \begin{pmatrix} 0 & \sqrt{m_1 m_2} & 0 \\ \sqrt{m_1 m_2} & m_2 & \sqrt{m_1 m_3} \\ 0 & \sqrt{m_1 m_3} & m_3 - m_1 \end{pmatrix} O = \begin{pmatrix} -m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{11}$$

with

$$\begin{aligned}
O &= \begin{pmatrix} \sqrt{\frac{m_2 m_3^2}{(m_2+m_1)(m_3^2-m_1^2)}} & \sqrt{\frac{m_1 m_3(m_3-m_2-m_1)}{(m_2+m_1)(m_3-m_2)(m_3-m_1)}} & \sqrt{\frac{m_1^2 m_2}{(m_3-m_2)(m_3^2-m_1^2)}} \\ -\sqrt{\frac{m_1 m_3}{(m_2+m_1)(m_3+m_1)}} & \sqrt{\frac{m_2(m_3-m_2-m_1)}{(m_2+m_1)(m_3-m_2)}} & \sqrt{\frac{m_1 m_3}{(m_3-m_2)(m_3+m_1)}} \\ \sqrt{\frac{m_1^2(m_3-m_2-m_1)}{(m_2+m_1)(m_3^2-m_1^2)}} & -\sqrt{\frac{m_1 m_2 m_3}{(m_3-m_2)(m_2+m_1)(m_3-m_1)}} & \sqrt{\frac{(m_3)^2(m_3-m_2-m_1)}{(m_3^2-m_1^2)(m_3-m_2)}} \end{pmatrix} \\
&\simeq \begin{pmatrix} 1 & \sqrt{\frac{m_1}{m_2}} & \sqrt{\frac{m_1^2 m_2}{m_3^3}} \\ -\sqrt{\frac{m_1}{m_2}} & 1 & \sqrt{\frac{m_1}{m_3}} \\ \sqrt{\frac{m_1^2}{m_2 m_3}} & -\sqrt{\frac{m_1}{m_3}} & 1 \end{pmatrix} \quad (\text{for } m_3 \gg m_2 \gg m_1). \tag{12}
\end{aligned}$$

The mass matrices for quarks, M_d and M_u are assumed to be of Type I as follows

$$M_d \simeq P_d \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b - m_d \end{pmatrix} P_d^\dagger, \quad M_u \simeq \begin{pmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & m_c & \sqrt{m_u m_t} \\ 0 & \sqrt{m_u m_t} & m_t - m_u \end{pmatrix} \tag{13}$$

where m_d , m_s and m_b are down quark masses and m_u , m_c and m_t are up quark masses. Those M_d and M_u are diagonalized by matrices $P_d O_d$ and O_u , respectively. Here orthogonal matrices O_d and O_u which diagonalize $P_d^\dagger M_d P_d$ and M_u are obtained from Eq. (22) by replacing m_1 , m_2 , m_3 by m_d , m_s , m_b and by m_u , m_c , m_t , respectively. In this case, the Cabbibo-Kobayashi-Maskawa (CKM) [4] quark mixing matrix V can be written as

$$V = P_q^{-1} P_d^{-1} O_u^T P_d O_d P_q \simeq \begin{pmatrix} |V_{11}| & |V_{12}| & |V_{13}| e^{-i\phi} \\ -|V_{12}| & |V_{22}| & |V_{23}| \\ |V_{12} V_{23}| - |V_{13}| e^{i\phi} & -|V_{23}| & |V_{33}| \end{pmatrix}. \tag{14}$$

where the P_d^{-1} factor is included to put V in the form with diagonal elements real to a good approximation. Furthermore, the P_q^{-1} and $P_q = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ with $\phi_1 - \phi_2 = \arg(P_d^{-1} O_u^T P_d O_d)_{12}$ and $\phi_1 - \phi_3 = \arg(P_d^{-1} O_u^T P_d O_d)_{23}$ are for the choice of phase convention as Eq. (14). The explicit forms and numerical center values of components of V are

$$|V_{12}| = \left| \sqrt{\frac{m_d(m_b + m_d)(m_b - m_s - m_d)}{(m_s + m_d)(m_b^2 - m_b m_s - m_d^2)}} - \sqrt{\frac{m_u(m_t + m_u)(m_t - m_c - m_u)}{(m_c + m_u)(m_t^2 - m_t m_c - m_u^2)}} e^{-i\alpha_{12}} \right|$$

$$\begin{aligned}
& -\sqrt{\frac{m_u^2(m_t^2 - m_u^2)(m_t - m_c - m_u)}{(m_c + m_u)(m_t^2 - m_t m_c - m_u^2)^2}} \sqrt{\frac{m_d(m_b - m_d)}{m_b^2 - m_b m_s - m_d^2}} e^{-i\alpha_{13}} \Big| \\
& \simeq \left| \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} e^{-i\alpha_{12}} \right| = 0.17 \sim 0.28, \\
|V_{23}| &= \left| \sqrt{\frac{m_u(m_t + m_u)(m_t - m_c - m_u)}{(m_c + m_u)(m_t^2 - m_t m_c - m_u^2)}} \sqrt{\frac{m_d^2 m_s}{(m_b - m_s)(m_b^2 - m_d^2)}} e^{i\alpha_{12}} \right. \\
& \quad \left. + \sqrt{\frac{m_d(m_b - m_d)}{m_b^2 - m_b m_s - m_d^2}} - \sqrt{\frac{m_u(m_t - m_u)}{m_t^2 - m_t m_c - m_u^2}} e^{-i\alpha_{23}} \right| \\
& \simeq \left| \sqrt{\frac{m_d}{m_b}} - \sqrt{\frac{m_u}{m_t}} e^{-i\alpha_{23}} \right| = 0.036 \sim 0.043, \\
|V_{13}| &= \left| \sqrt{\frac{m_d^2 m_s}{(m_b - m_s)(m_b^2 - m_d^2)}} + \sqrt{\frac{m_u^2(m_t^2 - m_u^2)(m_t - m_c - m_u)}{(m_c + m_u)(m_t^2 - m_t m_c - m_u^2)^2}} e^{-\alpha_{13}} \right. \\
& \quad \left. - \sqrt{\frac{m_u(m_t + m_u)(m_t - m_c - m_u)}{(m_c + m_u)(m_t^2 - m_t m_c - m_u^2)}} \sqrt{\frac{m_d(m_b - m_d)}{m_b^2 - m_b m_s - m_d^2}} e^{-i\alpha_{12}} \right| \\
& \simeq \left| \sqrt{\frac{m_d^2 m_s}{m_b^3}} - \sqrt{\frac{m_u}{m_c}} \left(\sqrt{\frac{m_d}{m_b}} - \sqrt{\frac{m_u}{m_t}} e^{-i\alpha_{23}} \right) e^{-i\alpha_{12}} \right| = 0.0021 \sim 0.0025, \\
\cos \phi & \simeq \frac{|V_{13}|^2 + |V_{12}|^2 |V_{23}|^2 - |V_{31}|^2}{2|V_{12}||V_{23}||V_{13}|} \simeq \frac{|V_{12}|^2 + m_u/m_c - m_d/m_s}{2|V_{12}|\sqrt{m_u/m_c}} = -1 \sim 1. \tag{15}
\end{aligned}$$

Here we have used the running quark mass at $\mu = m_Z$ [5]:

$$\begin{aligned}
m_u(m_Z) &= 2.33_{-0.45}^{+0.42} \text{MeV}, \quad m_c(m_Z) = 677_{-61}^{+56} \text{MeV}, \quad m_t(m_Z) = 181 \pm 13 \text{GeV}, \\
m_d(m_Z) &= 4.69_{-0.66}^{+0.60} \text{MeV}, \quad m_s(m_Z) = 93.4_{-13.0}^{+11.8} \text{MeV}, \quad m_b(m_Z) = 3.00 \pm 0.11 \text{GeV}. \tag{16}
\end{aligned}$$

Let us compare (15) with the experimental values [6]:

$$\begin{aligned}
|V_{12}|_{\text{exp}} &= 0.217 \sim 0.224, \quad |V_{23}|_{\text{exp}} = 0.036 \sim 0.042, \\
|V_{13}|_{\text{exp}} &= 0.0018 \sim 0.0045, \quad (90\% \text{CL}). \tag{17}
\end{aligned}$$

It is remarkable that the very heavy top quark mass raises no inconsistency in our model. The reason is as follows. In $|V_{23}|$, the first term of right-hand side in Eq. (15) ($\sqrt{m_d/m_b} = 0.034$) is nearly equal to the experimental value ($|V_{23}|_{\text{exp}} = 0.036 \sim 0.042$), so heavy top quark mass does not make any trouble. Whereas, in the case of Type II and also Fritzsch model [7], the first term of V_{23} becomes $\sqrt{m_s/m_b} = 0.18$. So, in order to adjust to the experimental value,

the second term must be of the same order as the first term to cancel a large part of the first term. Thus top quark could not have very heavy mass.

If we adopt only the central values of quark masses in Eq. (16), compatibility of our prediction Eq. (15) with the experimental values Eq. (17) imposes some constraints on α_{ij} . They are depicted in FIG. 1 in the shaded strip in α_{13} - α_{23} plane. In this figure we have superimposed the rephasing invariant Jarlskog parameter J of quark sector, $J = \text{Im}(V_{12}V_{22}^*V_{13}^*V_{23})$ [8]. However these restrictions are very sensitive to the errors of mass values and are not affirmative at least at this stage. Contours represent the value of J from -2.3×10^{-5} to 2.3×10^{-5} . The above restriction on α_{ij} , therefore, gives the bound on J as,

$$1.6 \times 10^{-5} \lesssim |J| \lesssim 2.2 \times 10^{-5}. \quad (18)$$

Using the popular approximation due to Wolfenstein [9], the CKM quark mixing matrix can be written in terms of only four real parameters:

$$\begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (19)$$

The measurement of the ρ and η parameters is usually associated to the determination of the only unknown vertex of a triangle in the $\rho - \eta$ plane whose other two vertices are in (0,0) and (1,0). This triangle is called the unitarity triangle. Changing freely α_{13} and α_{23} in Eq. (15), the predicted points sweep out light and dark gray regions (FIG. 2).

Next let us discuss the lepton sector. We develop our arguments first without seesaw mechanism. The mass matrix of leptons are assumed to be of Type II. Assigning $B = m_1$ and $D = m_3 + m_2$ (Type II) in Eq. (8), we obtain from Eq. (7)

$$A = \sqrt{\frac{m_1(-m_2)m_3}{m_3 + m_2}}, \quad C = \sqrt{\frac{(-m_2)m_3(m_3 + m_2 - m_1)}{m_3 + m_2}}. \quad (20)$$

Then, we obtain the mass matrix M of Type II and the orthogonal matrix O which diagonalize it, which are expressed in terms of mass eigen value m_i as

$$\begin{aligned}
M &= \begin{pmatrix} 0 & \sqrt{\frac{m_1 m_2 m_3}{m_3 - m_2}} & 0 \\ \sqrt{\frac{m_1 m_2 m_3}{m_3 - m_2}} & m_1 & \sqrt{\frac{m_2 m_3 (m_3 - m_2 - m_1)}{m_3 - m_2}} \\ 0 & \sqrt{\frac{m_2 m_3 (m_3 - m_2 - m_1)}{m_3 - m_2}} & m_3 - m_2 \end{pmatrix} \\
&\simeq \begin{pmatrix} 0 & \sqrt{m_1 m_2} & 0 \\ \sqrt{m_1 m_2} & m_1 & \sqrt{m_2 m_3} \\ 0 & \sqrt{m_2 m_3} & m_3 - m_2 \end{pmatrix}, \tag{21}
\end{aligned}$$

$$\begin{aligned}
O &= \begin{pmatrix} \sqrt{\frac{m_2 m_3 (m_3 - m_2 - m_1)}{(m_1 + m_2)(m_3 - m_1)(m_3 - m_2)}} & \sqrt{\frac{m_1 m_3^2}{(m_1 + m_2)(m_3^2 - m_2^2)}} & \sqrt{\frac{m_2^2 m_1}{(m_3 - m_1)(m_3^2 - m_2^2)}} \\ \sqrt{\frac{m_1 (m_3 - m_2 - m_1)}{(m_1 + m_2)(m_3 - m_1)}} & -\sqrt{\frac{m_2 m_3}{(m_1 + m_2)(m_3 + m_2)}} & \sqrt{\frac{m_2 m_3}{(m_3 - m_1)(m_3 + m_2)}} \\ -\sqrt{\frac{m_2 m_1 m_3}{(m_3 - m_1)(m_1 + m_2)(m_3 - m_2)}} & \sqrt{\frac{m_2^2 (m_3 - m_1 - m_2)}{(m_1 + m_2)(m_3^2 - m_2^2)}} & \sqrt{\frac{(m_3)^2 (m_3 - m_1 - m_2)}{(m_3 - m_1)(m_3^2 - m_2^2)}} \end{pmatrix} \\
&\simeq \begin{pmatrix} 1 & \sqrt{\frac{m_1}{m_2}} & \sqrt{\frac{m_1 m_2^2}{m_3^3}} \\ \sqrt{\frac{m_1}{m_2}} & -1 & \sqrt{\frac{m_2}{m_3}} \\ -\sqrt{\frac{m_1}{m_3}} & \sqrt{\frac{m_2}{m_3}} & 1 \end{pmatrix} \quad (\text{for } m_3 \gg m_2 \gg m_1) \tag{22}
\end{aligned}$$

with

$$O^T M O = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & -m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{23}$$

where we have transformed m_2 into $-m_2$. The component (2,3) and (3,2) of O is not small comparing with $\sqrt{m_1/m_3}$ in Type I. Therefore, due to this large mixing, Type II can be consistent with the large ν_μ - ν_τ mixing angle solution in atmospheric neutrino experiment as shown later.

The mass matrices of charged leptons and neutrinos are assumed to be of Type II as follows

$$M_e \simeq P_e \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_e & \sqrt{m_\mu m_\tau} \\ 0 & \sqrt{m_\mu m_\tau} & m_\tau - m_\mu \end{pmatrix} P_e^\dagger, \quad M_\nu \simeq \begin{pmatrix} 0 & \sqrt{m_1 m_2} & 0 \\ \sqrt{m_1 m_2} & m_1 & \sqrt{m_2 m_3} \\ 0 & \sqrt{m_2 m_3} & m_3 - m_2 \end{pmatrix}, \tag{24}$$

where m_e , m_μ and m_τ are charged lepton masses and m_1 , m_2 and m_3 are neutrino masses. Those M_e and M_ν are diagonalized by matrices $P_e O_e$ and O_ν , respectively. Here orthogonal matrix O_ν is obtained from Eq. (22) with taking m_i as neutrino mass and O_e by replacing m_1, m_2, m_3 by m_e, m_μ, m_τ . In this case, lepton mixing matrix U (hereafter we call it the Maki-Nakagawa-Sakata (MNS) mixing matrix [11]), is given by

$$U = P_l^\dagger P_e^\dagger O_e^T P_e O_\nu P_l = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}, \quad (25)$$

where $P_l = \text{diag}(1, i, 1)$ is included to have positive neutrino mass. $P_l^\dagger P_e^\dagger$ factor leads U to the form whose diagonal elements are real to a good approximation. We obtain the expressions of some elements of U as follows,

$$\begin{aligned} U_{12} &\simeq i \left(\sqrt{\frac{m_1}{m_2}} - \sqrt{\frac{m_e}{m_\mu}} e^{i\beta_{12}} \right), & U_{23} &\simeq -i \left(-\sqrt{\frac{m_2}{m_3}} + \sqrt{\frac{m_\mu}{m_\tau}} e^{i\beta_{23}} \right), \\ U_{13} &\simeq \sqrt{\frac{m_e}{m_\mu}} e^{i\beta_{12}} \left(\sqrt{\frac{m_2}{m_3}} - \sqrt{\frac{m_\mu}{m_\tau}} e^{i\beta_{23}} \right). \end{aligned} \quad (26)$$

For example, substituting the neutrino masses,

$$m_1 = 1.4 \times 10^{-4} \text{eV}, \quad m_2 = 3.2 \times 10^{-3} \text{eV}, \quad m_3 = 7.1 \times 10^{-2} \text{eV}, \quad (27)$$

and the charged lepton masses, $m_e = 0.51 \text{MeV}$, $m_\mu = 106 \text{MeV}$, $m_\tau = 1.77 \text{GeV}$, into Eqs. (25) we obtain

$$|U_{12}| = 0.14 \sim 0.28, \quad |U_{23}| = 0.033 \sim 0.46, \quad |U_{13}| = 0.023 \sim 0.032. \quad (28)$$

Here we have used $\Delta m_{\text{atm}} = m_3^2 - m_2^2 = 5.0 \times 10^{-3} \text{eV}^2$ and $\Delta m_{\text{solar}} = m_2^2 - m_1^2 = 1.0 \times 10^{-5} \text{eV}^2$ with the assumption that $m_1 \ll m_2 \ll m_3$ and $m_1/m_2 = m_2/m_3$. Let us compare this prediction with the experimental values [12]:

$$|U_{12}|_{\text{exp}} = 0 \sim 0.71, \quad |U_{23}|_{\text{exp}} = 0.52 \sim 0.87, \quad |U_{13}|_{\text{exp}} = 0 \sim 0.22. \quad (29)$$

Here we have combined the constraints from the recent CHOOZ reactor experiment [13] and the Super KAMIOKANDE atmospheric neutrino experiment [14].

Though the lepton mass matrices M_e and M_ν of Type II lead to large ν_μ - ν_τ mixing, $|U_{23}|$ is still small compared with the experimental value. This trouble is resolved via seesaw mechanism. In the seesaw mechanism, we have additional free parameters even in our model. So we set the following assumptions guided by the atmospheric neutrino oscillation experiments, which lead to a fairly large ν_μ - ν_τ mixing.

- (a) Mass matrices M_e , M_D and M_R belong to Type I, instead of Type II, similarly to quark mass matrices.
- (b) Mass eigen values of M_D and M_R satisfy

$$m_{D3} : m_{D2} : m_{D1} = 1 : x : x^2, \quad (30)$$

$$m_{R3} : m_{R2} : m_{R1} = 1 : x^2 : x^3. \quad (31)$$

Here m_{Di} and m_{Ri} are eigen values of M_D and M_R , respectively, and x is a small parameter.

It is noted from assumption (a) that M_ν itself is out of Type I via seesaw mechanism. If we use the assumption that M_e , M_D and M_R belong to Type II instead of Type I, we can not accommodate m_{R3} , m_{R2} and m_{R1} to a large ν_μ - ν_τ mixing. Conversely, a large mixing enforces us m_{R1} and m_{R2} of the same order, where we can not distinguish Type II from Type I.

Using assumptions (a) and (b), we obtain

$$M_D \stackrel{(a)}{=} \begin{pmatrix} 0 & \sqrt{\frac{m_{D1}m_{D2}m_{D3}}{m_{D3}-m_{D1}}} & 0 \\ \sqrt{\frac{m_{D1}m_{D2}m_{D3}}{m_{D3}-m_{D1}}} & m_{D2} & \sqrt{\frac{m_{D1}m_{D3}(m_{D3}-m_{D2}-m_{D1})}{m_{D3}-m_{D1}}} \\ 0 & \sqrt{\frac{m_{D1}m_{D3}(m_{D3}-m_{D2}-m_{D1})}{m_{D3}-m_{D1}}} & m_{D3} - m_{D1} \end{pmatrix} \quad (32)$$

$$\stackrel{(b)}{\simeq} m_{D3} \begin{pmatrix} 0 & x\sqrt{x} & 0 \\ x\sqrt{x} & x & x \\ 0 & x & 1 \end{pmatrix} \quad (33)$$

and similarly,

$$M_R \simeq m_{R3} \begin{pmatrix} 0 & x^2\sqrt{x} & 0 \\ x^2\sqrt{x} & x^2 & x\sqrt{x} \\ 0 & x\sqrt{x} & 1 \end{pmatrix}. \quad (34)$$

Then the neutrino mass matrix M_ν is given by

$$M_\nu = -M_D^T M_R^{-1} M_D = -\frac{(m_{D3})^2}{m_{R3}} \begin{pmatrix} 0 & \sqrt{x} & 0 \\ \sqrt{x} & 1 + (\sqrt{x} - x)^2 & 1 - (\sqrt{x} - x) \\ 0 & 1 - (\sqrt{x} - x) & 1 \end{pmatrix}. \quad (35)$$

The orthogonal matrix which diagonalizes Eq. (35) is

$$O_\nu \simeq \begin{pmatrix} -\frac{1}{12} \frac{-72+48\sqrt{3}-9\sqrt{x}+5\sqrt{3}\sqrt{x}}{(\sqrt{3}-1)(3-\sqrt{3})^{3/2}} & \frac{1}{12} \frac{72+48\sqrt{3}+5\sqrt{3}\sqrt{x}+9\sqrt{x}}{(1+\sqrt{3})(3+\sqrt{3})^{3/2}} & \frac{\sqrt{2}}{4}x^{1/2} + \frac{\sqrt{2}}{8}x \\ \frac{1}{24} \frac{-72+48\sqrt{3}+21\sqrt{x}-7\sqrt{3}\sqrt{x}}{(3-\sqrt{3})^{3/2}} & \frac{1}{24} \frac{72+48\sqrt{3}-21\sqrt{x}-7\sqrt{3}\sqrt{x}}{(3+\sqrt{3})^{3/2}} & \frac{\sqrt{2}}{2} + \frac{5\sqrt{2}}{32}x \\ -\frac{1}{24} \frac{-72+48\sqrt{3}-15\sqrt{x}+5\sqrt{3}\sqrt{x}}{(3-\sqrt{3})^{3/2}} & -\frac{1}{24} \frac{72+48\sqrt{3}+15\sqrt{x}+5\sqrt{3}\sqrt{x}}{(3+\sqrt{3})^{3/2}} & \frac{\sqrt{2}}{2} - \frac{7\sqrt{2}}{32}x \end{pmatrix}, \quad (36)$$

And the eigen mass is

$$\begin{aligned} m_1 &\simeq \frac{m_{D3}^2}{m_{R3}} \left\{ \left(\frac{1}{2} - \frac{\sqrt{3}}{2} \right) \sqrt{x} - \left(\frac{3}{8} - \frac{\sqrt{3}}{24} \right) x \right\}, \\ m_2 &\simeq \frac{m_{D3}^2}{m_{R3}} \left\{ \left(\frac{1}{2} + \frac{\sqrt{3}}{2} \right) \sqrt{x} - \left(\frac{3}{8} + \frac{\sqrt{3}}{24} \right) x \right\}, \quad m_3 \simeq \frac{m_{D3}^2}{m_{R3}} \left\{ 2 - \sqrt{x} + \frac{7}{4}x \right\}. \end{aligned} \quad (37)$$

For numerical estimation we assume that mass pattern Eq. (30) is same as that of up quark.

$$m_t(m_Z) : m_c(m_Z) : m_u(m_Z) = 1 : x : x^2, \quad (x \simeq 0.0036) \quad (38)$$

and, therefore, $m_{D3} = k \times m_t(m_Z)$, $m_{D2} = kx \times m_t(m_Z)$ and $m_{D1} = kx^2 \times m_t(m_Z)$. Using the assumption (a) that M_e belong to type I, the mass ratios of light Majorana neutrinos, the MNS matrix U and the rephasing invariant Jarlskog parameter J of lepton sector become

$$m_3 : m_2 : -m_1 \simeq 1.0 : 0.04 : 0.01, \quad (39)$$

$$\begin{aligned} U &= P_e^\dagger O_e^T P_e O_\nu \\ &\simeq \begin{pmatrix} -0.88 - 0.02e^{-i\beta_{12}} & 0.46 - 0.04e^{-i\beta_{12}} & 0.022 - 0.049e^{-i\beta_{12}} \\ 0.34 - 0.06e^{i\beta_{12}} & 0.62 + 0.03e^{i\beta_{12}} + 0.01e^{-i\beta_{23}} & 0.71 - 0.01e^{-i\beta_{23}} \\ -0.31 + 0.01e^{i\beta_{23}} & -0.64 + 0.01e^{i\beta_{23}} & 0.71 + 0.01e^{i\beta_{23}} \end{pmatrix} \end{aligned} \quad (40)$$

and $|J| \lesssim 0.01$. Here we have assumed that the changes of lepton masses and the MNS mixing from $\mu = m_Z$ to $\mu = \text{MeV}$ are very small. At this stage only one parameter, m_{R3} , still remains free. It will be determined from $\Delta m_{32}^2 = 5.0 \times 10^{-3} \text{eV}^2$ as

$$m_{R3} = k^2 \times (9.0 \times 10^{23}) \text{eV}. \quad (41)$$

Thus we have fixed parameters so as to adjust the atmospheric neutrino oscillation experiments. The assumptions (a) and (b) are not unique and their justification is checked by the compatibility with the solar neutrino deficit experiments. From Eqs. (37), (40) and (41), we have the restrictive prediction.

$$\Delta m_{21}^2 \simeq 7.8 \times 10^{-6} \text{eV}^2, \quad \tan^2 \varphi \equiv \frac{|U_{13}|^2}{|U_{23}|^2 + |U_{33}|^2} \simeq 0, \quad \tan^2 \omega \equiv \frac{|U_{12}|^2}{|U_{11}|^2} \simeq 0.27, \quad (42)$$

which are superimposed on the analyses by Fogli et. al. [15] (FIG. 3). The star indicates our prediction. The position of star has been determined from the atmospheric neutrino experiments and was free from the solar neutrino deficit experiments. Nevertheless its position in the allowed region of solar neutrino experiments.

Conclusive remarks are in order. We started with the same type of 4 texture zero mass matrices both for quarks and leptons. They were classified into Type I and II. Type I explains quark sector consistently. For the lepton sector Type II, on the other hand, reproduces qualitatively large lepton mixing. However, best fitting with experimental data requires the seesaw mechanism in lepton sector with Type I mass matrices similarly to quarks.

Acknowledgements

We are grateful to H.Minakata and O.Yasuda for valuable discussions. Special thanks are due to Y. Koide for his enlightening suggestions on phenomenological mass matrix models.

REFERENCES

- [1] See for the zero texture, for example, P.Ramond, R.G.Roberts and G.G.Ross: Nucl.Phys. **B406** 19 (1993).
- [2] T.Yanagida: Proceedings of the Workshop on the unified theory and baryon number in the Universe, eds. O.Sawada and A.Sugamoto (KEK, Tsukuba)95 (1979); M.Gell-Mann, P.Ramond and R.Slansky: Supergravity, eds. P.van Nieuwenhuizen and D.Freedman (North-Holland, Amsterdam) 315 (1979); G.Senjanović and R.N.Mohapatra: Phys.Rev.Lett. **44** 912 (1980).
- [3] G.C.Branco, L.Lavoura and F.Mota: Phys.Rev. **D39** 3443 (1989).
- [4] N.Cabibbo: Phys.Rev.Lett. **10** 531 (1963); M.Kobayashi and T. Maskawa: Prog. Theor. Phys. **49** 652 (1973).
- [5] H.Fusaoka and Y.Koide: Phys.Rev. **D57** 3986 (1998).
- [6] C. Caso et. al. (Particle Data Group): The European Physical Journal **C3** 1 (1998).
- [7] H.Fritzsch: Nucl.Phys. **B155** 189 (1979).
- [8] C.Jarlskog: Phys.Rev.Lett. **55** 1839 (1985).
- [9] L.Wolfenstein: Phys.Rev.Lett. **51** 1945 (1983).
- [10] J.L.Rosner: Nucl.Instrum.Meth. **A408** 308 (1998).
- [11] Z.Maki, M.Nakagawa and S.Sakata: Prog.Theo.Phys. **28** 247 (1962).
- [12] T.Fukuyama, K.Matsuda, H.Nishiura: Mod.Phys.Lett. **A13** 2279 (1998).
- [13] M.Apollonio et.al. Phys.Lett. **B420** 397 (1998).
- [14] T.Kajita, talk at Neutrino '98 (Takayama, Japan, June 1998).
- [15] G.L.Fogli, E.Lisi, D.Montanino and G.Scioscia: Phys.Rev. **D55** 4385 (1997).

FIGURES

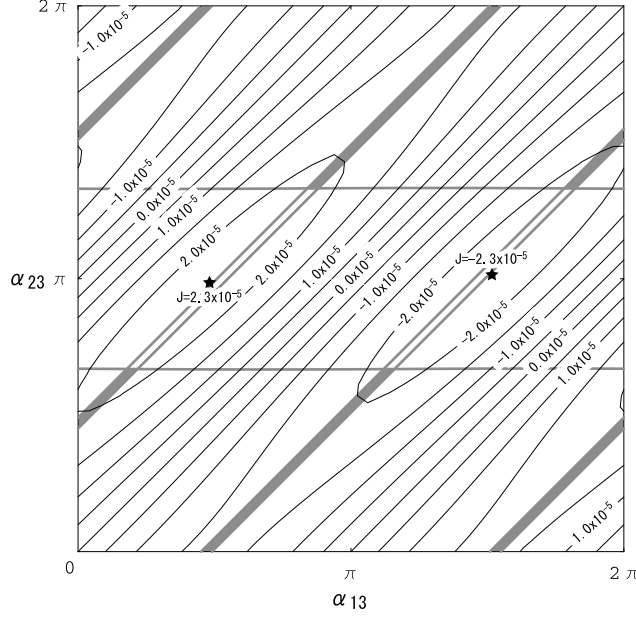


FIG. 1. The allowed region on α_{13} - α_{23} plane is depicted by the shaded areas. In the allowed region, the contours indicate the rephasing invariant of Jarlskog parameter $J(\equiv \text{Im}(V_{12}V_{22}^*V_{13}^*V_{23}))$ of quark sector.

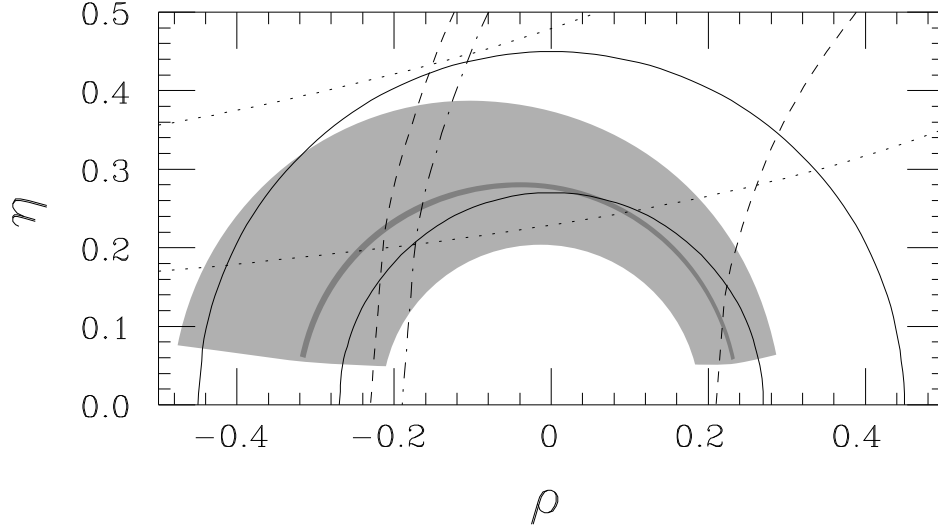


FIG. 2. The vertex position of unitarity triangle predicted by our model is superimposed on the diagram restricted by hadron experiments. Our predictions is obtained by changing α_{13} and α_{23} freely in Eq. (15) with no approximation. If each quark mass takes the center values in Eqs. (16), the dark gray region is allowed. On the other hand, taking the error of each quark mass into consideration, we obtain the light gray region.

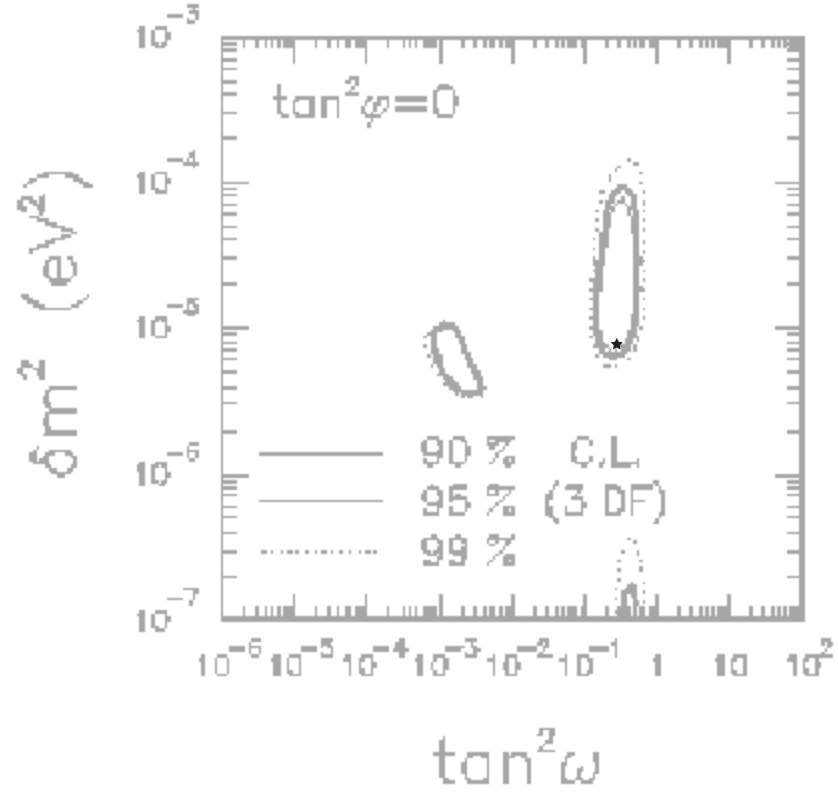


FIG. 3. The solid line and dotted line show 90% C.L. and 99% C.L., respectively, which were derived from the three-flavor analysis of the solar neutrino deficit experiments [15]. The star indicates our prediction.